

measurement of the longitudinal polarization of the beta particles will establish the cancellation effect in this decay because, in such a case, the longitudinal polarization will be substantially different from the  $v/c$  law.

*Note added in the proof.* After this work was sent for publication, a similar work has been reported by Fischbeck and Newsome in *Bull. Am. Phys. Soc.* **8**, 332

(1963). Their measurements are in very good agreement with our results.

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## Octupole Deformation in Even-Even Medium Mass Nuclei

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The stability of octupole deformation for even-even medium mass nuclei with small spheroidal deformation has been studied in order to explain the existence of odd-parity excited states in some of them. The total single-particle energy is calculated by an exact diagonalization of the Nilsson Hamiltonian with octupole deformation, neglecting the residual two-particle interaction. Within this framework, it is found that these nuclei are stable against octupole deformation.

IN the past years much attention has been given to the experimental investigation of excited states of even-even medium mass nuclei. A systematic occurrence of low-lying odd-parity excited states with spin 3- or 5- in a number of even-even nuclei, with mass number in the range  $60 \leq A \leq 150$ , has been observed recently by various workers.<sup>1-5</sup> Most of these odd-parity excited states lie within the energy range of 2 to 3 MeV.

It is known that the nuclei in the region under consideration have spectra of a vibrational kind. These spectra have been interpreted in various ways,<sup>6-11</sup> but the most widely held view is that outside the rotational regions and excluding the few closed-shell nuclei, there are nuclei which have a tendency to deform, but the deformation has fluctuations large compared with the magnitude of the deformation and since these fluctuations in shape have dynamical properties there should

be nuclear excitation analogous to waves on the nuclear surface. The known spectra have been interpreted in terms of quadrupole surface vibrations. Since the expressions for the reduced transition rate in the vibration and rotation models are identical, one can assign a mean deformation to these nuclei.<sup>12</sup>

The occurrence of the odd-parity excited state in these nuclei has given rise to the question of the existence of octupole deformation in them. The purpose of the present work is to see whether such a deformation is energetically favored for some nuclei in this range. The stability of octupole deformation in some nuclei lying in the rare-earth and actinide region and possessing a particular value of spheroidal deformation have been studied earlier by Dutt and Mukherjee.<sup>13</sup> But the question of the stability of octupole deformation for nuclei lying in the region under consideration and having different spheroidal deformation has not received attention.

We have calculated the total single-particle energy by an exact diagonalization of Nilsson<sup>14</sup> Hamiltonian with an additional term for the octupole deformation. The Hamiltonian we have used may, therefore, be written as

$$H = \chi \hbar \omega_0^0 \left\{ \frac{2K}{a_2} \left( \frac{1}{2} \nabla^2 - \frac{1}{2} r^2 \right) - Kr^2 \left[ 9 \left( \frac{3}{140\pi} \right)^{1/2} a_3 Y_1^0(\theta, \phi) + Y_2^0(\theta, \phi) + \frac{a_3}{a_2} Y_3^0(\theta, \phi) \right] - 2\mathbf{L} \cdot \mathbf{s} - \mu \mathbf{L}^2 \right\}, \quad (1)$$

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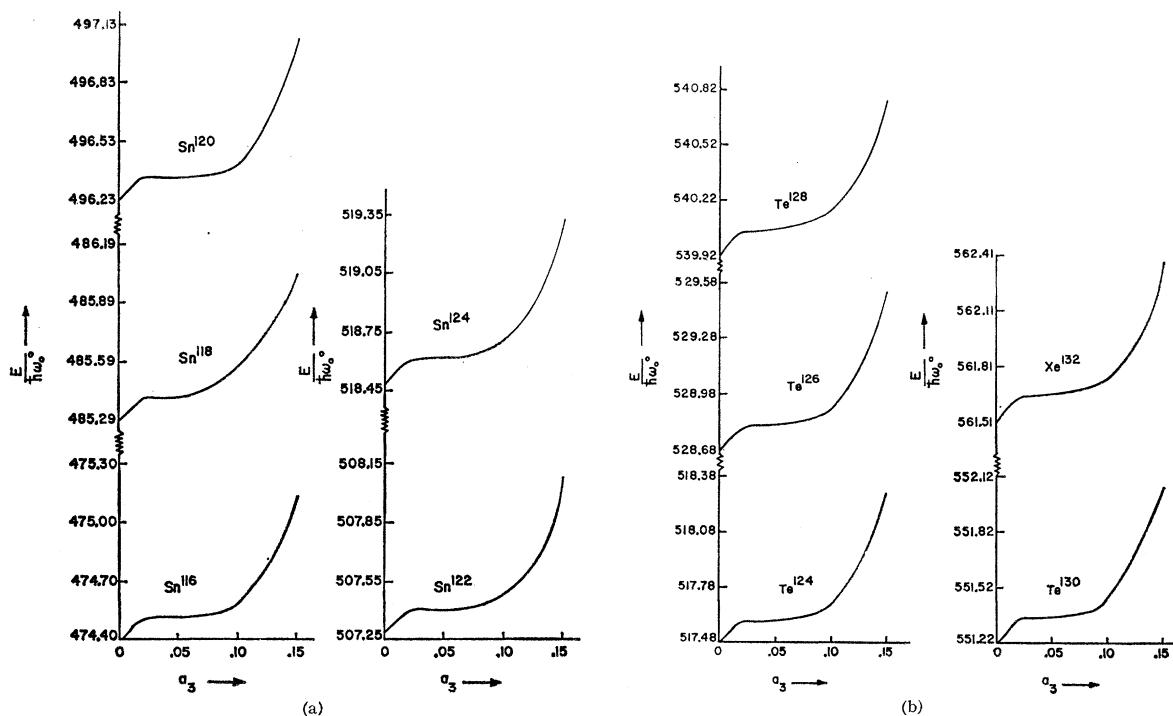


FIG. 1. The total single-particle energies of nuclei as a function of octupole deformation parameter  $a_3$ .

where  $a_2$  and  $a_3$  are the spheroidal and octupole deformation parameters, respectively, and the other symbols have their usual meaning. The parameters  $\chi=0.05$  and  $\mu=0.35$  to  $0.45$  are taken from Nilsson,<sup>14</sup> and

$$K = \begin{bmatrix} 4 \left( \frac{\pi}{5} \right)^{1/2} & 100 \\ -\frac{3}{5} & \eta - \frac{a_2 a_3^2}{64\pi} \end{bmatrix}. \quad (2)$$

Most of the nuclei, considered here, have roughly a stable spheroidal deformation,<sup>12</sup>  $a_2 = -0.26$ . This value of  $a_2$  corresponds to  $\eta = 2.5$  and  $K = (2.64225 + 0.13085a_3^2)$ . As in the works of Lee and Inglis,<sup>15,16</sup> the residual two-particle interaction is not considered and it is hoped that the essential conclusion will remain unaffected even after its inclusion.

The only good quantum number for the above Hamiltonian is  $j_z$ , hence, the wave function is represented by

$$|j_z\rangle = \sum_{N11_2s_z} a_{N11_2s_z} |N11_2s_z\rangle, \quad (3)$$

where  $|N11_2s_z\rangle$  are the wave functions corresponding to the spherical limit ( $a_2 = a_3 = 0$ ). The matrix elements of

$H$  for states up to  $N=6$  and  $j_z = 11/2, 9/2, 7/2, 5/2, 3/2$ , and  $1/2$  have been calculated for different values of  $a_3$  ranging from 0 to 0.15. The highest order matrix encountered in the calculation is  $28 \times 28$  for  $j_z = 1/2$ . The exact diagonalization of the matrices is carried out at the Computing Center of the University of Southern California.

The total single-particle energy, expressed in units of  $\hbar\omega_0^0$ , for isotopes of Sn, Te, and Xe, which roughly possess a spheroidal deformation  $a_2 = -0.26$ , have been plotted against the octupole deformation parameter  $a_3$  in Figs. 1(a) and 1(b). The total single-particle energy for each nucleus has been calculated by considering those levels for which the total energy is minimum for each value of  $a_3$ . It is evident from the figures that the total single-particle energy increases with increasing value of  $a_3$  for all the nuclei under consideration and the nature of variation is nearly the same in all of them. Therefore, it is concluded that within the framework of the Nilsson model considered here, the isotopes of Sn and Te are stable against octupole deformation and the existence of a 3-state in Sn<sup>116</sup> and Te<sup>124</sup> still remains an open question.

One of the authors (MLR) is grateful to R. Crowley for help with the computing work.

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